Conic Sections

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Mathematical formulation of a general conic section is given by

$$\boxed{\rho = \frac{l}{1-\epsilon\cos\phi} \Rightarrow \sqrt{x^2+y^2} = (l+\epsilon x)}$$

$$\boxed{\Rightarrow \left(\frac{x-\frac{l\epsilon}{(1-\epsilon^2)}}{\frac{l}{1-\epsilon^2}}\right)^2 + \left(1-\epsilon^2\right)\frac{y^2}{l^2} = 1}$$

For the special case of $\epsilon = 1$, i.e., parabola,

$$\boxed{y^2 = l^2 + 2lx}$$

conic section	equation	eccentricity (ϵ)	linear eccentricity(c)	semi-latus rectum(l)	focal parameter(p)
circle	$x^2+y^2=R_0^2$	0	0	r	∞
ellipse	$\frac{\left(x-\sqrt{a^2-b^2}\right)^2}{a^2}+\frac{y^2}{b^2}=1$	$\frac{\sqrt{a^2-b^2}}{a}$	$+\sqrt{a^2-b^2}$	$\frac{b^2}{a}$	$\frac{b^2}{\sqrt{a^2-b^2}}$
parabola	$y^2=4a\,(x+a)$	1	$-a$	$2a$	$2a$
hyperbola	$\frac{\left(x+\sqrt{a^2+b^2}\right)^2}{a^2}-\frac{y^2}{b^2}=1$	$\frac{\sqrt{a^2+b^2}}{a}$	$-\sqrt{a^2+b^2}$	$\frac{b^2}{a}$	$\frac{b^2}{\sqrt{a^2+b^2}}$

- Focal points of all the above conic sections are placed at the ***origin ...***
- For ellipse, left focal point is placed at the origin, where as right focal point is placed at the origin for hyperbola ...
- For, parabola only one simple focal point ... so, no problem !!!

Definitions

- The latus rectum (2ℓ) is the chord parallel to the directrix and passing through the focus (or one of the two foci).
- The semi-latus rectum (ℓ) is half the latus rectum.

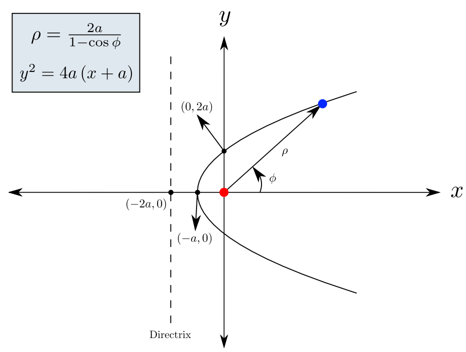
- The focal parameter is the distance from the focus (or one of the two foci) to the directrix.

$$p = \frac{l}{\epsilon}$$

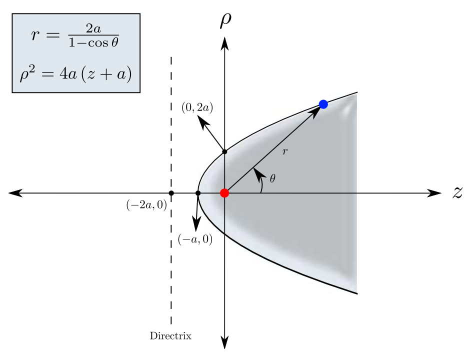
- The linear eccentricity (c) is the x-axis offset in positive direction ... (shifting of conic section along the x-direction so that its focal-point is now at the origin)

$$c = \left(\frac{l\epsilon}{1 - \epsilon^2} \right)$$

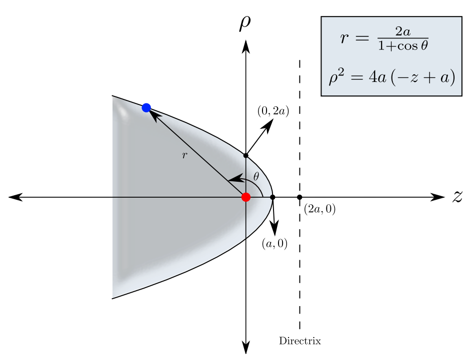
Parabola ++



(a) Simple parabola

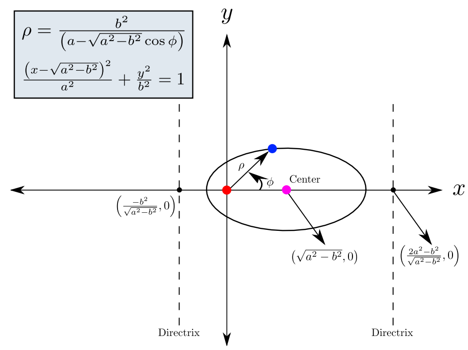


(b) Praboloid of revolution opening along positive z-axis

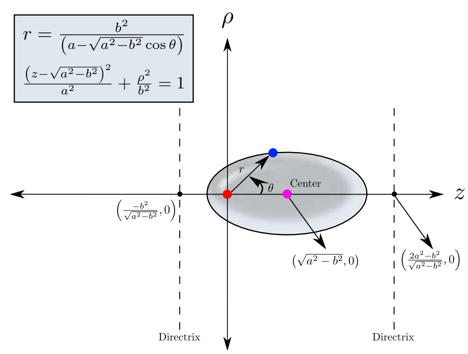


(c) Praboloid of revolution opening along negative z-axis

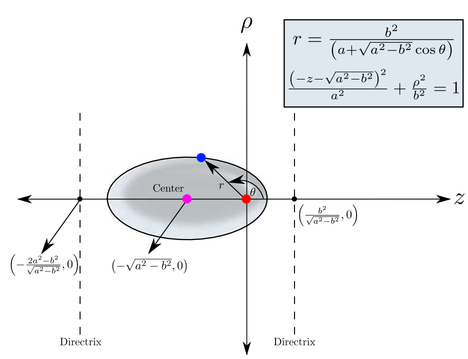
Ellipse ++



(d) Simple ellipse

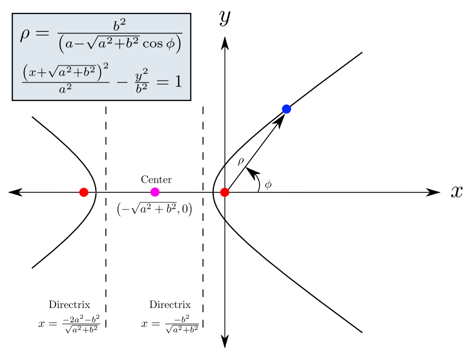


(e) Ellipsoid of revolution along positive z-axis

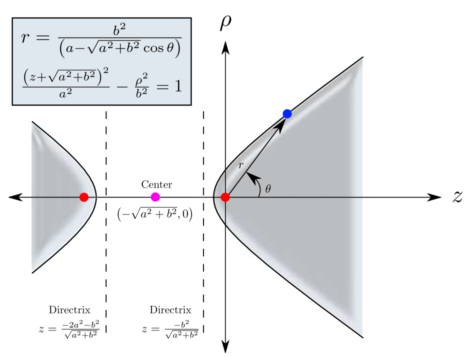


(f) Ellipsoid of revolution along negative z-axis

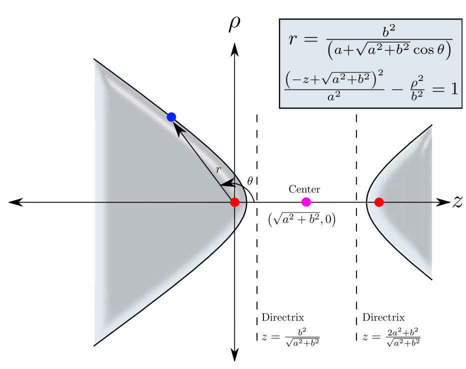
Hyperbola ++



(g) Simple hyperbola



(h) Hyperboloid of revolution along positive z-axis



(i) Hyperboloid of revolution along negative z-axis