Array Antennas - Theory & Design

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Outline

1. Array Factor
2. Linear Arrays
3. Linear Arrays - Examples
4. Planar Arrays
5. Planar Arrays - Examples
6. Synthesis
7. Multiple Beam Arrays
8. Problems
Outline

1. Array Factor
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Cylindrical and Spherical Coordinate Systems

\[ \vec{r} = x\hat{x} + y\hat{y} + z\hat{z} \]

\[ \vec{r} = \rho \cos \phi \hat{x} + \rho \sin \phi \hat{y} + z\hat{z} \]

\[ \vec{r} = r \sin \theta \cos \phi \hat{x} + r \sin \theta \sin \phi \hat{y} + r \cos \theta \hat{z}, \]
Array Factor

\[
AF = \sum_n A_n \exp\left[jk_0 \left( |\vec{r}| - |\vec{r} - \vec{r}_n| \right) \right]
\]
Approximation of $| \vec{r} - \vec{r}' |$ - Cartesian System

In Cartesian coordinate system,

\[
\vec{r'} = x' \hat{x} + y' \hat{y} + z' \hat{z}, \quad \text{and} \\
\vec{r} = r \sin \theta \cos \phi \hat{x} + r \sin \theta \sin \phi \hat{y} + r \cos \theta \hat{z}.
\]

So,

\[
| \vec{r} - \vec{r'} | = | r \sin \theta \cos \phi \hat{x} + r \sin \theta \sin \phi \hat{y} + r \cos \theta \hat{z} - x' \hat{x} - y' \hat{y} - z' \hat{z} |
\]

\[
= \sqrt{(r \sin \theta \cos \phi - x')^2 + (r \sin \theta \sin \phi - y')^2 + (r \cos \theta - z')^2}
\]

\[
= \sqrt{r^2 - 2rx' \sin \theta \cos \phi - 2ry' \sin \theta \sin \phi - 2rz' \cos \theta + x'^2 + y'^2 + z'^2}
\]

\[
\approx \sqrt{r^2 - 2rx' \sin \theta \cos \phi - 2ry' \sin \theta \sin \phi - 2rz' \cos \theta}
\]

\[
= r \sqrt{1 - \frac{2x' \sin \theta \cos \phi + 2y' \sin \theta \sin \phi + 2z' \cos \theta}{r}}
\]

\[
\approx r \left( 1 - \frac{1}{2} \frac{2x' \sin \theta \cos \phi + 2y' \sin \theta \sin \phi + 2z' \cos \theta}{r} \right)
\]

\[
= \left[ r - \left( x' \sin \theta \cos \phi + y' \sin \theta \sin \phi + z' \cos \theta \right) \right].
\]

(1)
Approximation of $|\vec{r} - \vec{r}'|$ - Cylindrical System

In Cylindrical coordinate system,

$$\vec{r}' = \rho' \cos \phi' \hat{x} + \rho' \sin \phi' \hat{y} + z' \hat{z}, \text{ and}$$

$$\vec{r} = r \sin \theta \cos \phi \hat{x} + r \sin \theta \sin \phi \hat{y} + r \cos \theta \hat{z}.$$

So, following the same procedure given in the previous slide

$$|\vec{r} - \vec{r}'| \approx r - (x' \sin \theta \cos \phi + y' \sin \theta \sin \phi + z' \cos \theta)$$
$$= r - (\rho' \cos \phi' \sin \theta \cos \phi + \rho' \sin \phi' \sin \theta \sin \phi + z' \cos \theta)$$
$$= r - (\rho' \sin \theta (\cos \phi' \cos \phi + \sin \phi' \sin \phi) + z' \cos \theta)$$
$$= r - (\rho' \sin \theta \cos (\phi - \phi') + z' \cos \theta).$$  \hspace{2cm} (2)
Approximation of $|\vec{r} - \vec{r}'|$ - Spherical System ***

In Spherical coordinate system,

$$\vec{r}' = r' \sin \theta' \cos \phi' \hat{x} + r' \sin \theta' \sin \phi' \hat{y} + r' \cos \theta' \hat{z}, \text{ and}$$

$$\vec{r} = r \sin \theta \cos \phi \hat{x} + r \sin \theta \sin \phi \hat{y} + r \cos \theta \hat{z}.$$ 

So, following the same procedure given in the previous slide

$$|\vec{r} - \vec{r}'| \approx r - (x' \sin \theta \cos \phi + y' \sin \theta \sin \phi + z' \cos \theta)$$

$$= r - (r' \sin \theta' \cos \phi' \cos \theta' \sin \phi' \sin \theta + r' \cos \theta' \cos \theta)$$

$$= r - (r' \sin \theta \sin \theta' (\cos \phi' \cos \phi + \sin \phi' \sin \phi) + r' \cos \theta' \cos \theta)$$

$$= r - (r' \sin \theta \sin \theta' \cos (\phi - \phi') + r' \cos \theta' \cos \theta). \quad (3)$$
So, Array Factor in Cartesian Co-ordinate System is ...

\[
AF = \sum_n A_n \exp \left[ jk_0 \left( |\vec{r}| - |\vec{r} - \vec{r}_n| \right) \right]
\]

\[
\approx \sum_n A_n \exp \left\{ jk_0 \left[ |\vec{r}| - \left( \vec{r} - \left( x'_n \sin \theta \cos \phi + y'_n \sin \theta \sin \phi + z'_n \cos \theta \right) \right) \right\}
\]

\[
= \sum_n A_n \exp \left( jk_0 \left( x'_n \sin \theta \cos \phi + y'_n \sin \theta \sin \phi + z'_n \cos \theta \right) \right)
\]

\[
= \sum_n A_n \exp \left( jk_0 \sin \theta \cos \phi x'_n + jk_0 \sin \theta \sin \phi y'_n + jk_0 \cos \theta z'_n \right)
\]

\[
= \sum_n A_n \exp \left( jk_0 x'_n + jk_0 y'_n + jk_0 z'_n \right)
\]

(4)

For continuous arrays, the above equation reduces to

\[
AF = \iiint A(x', y', z') \exp \left( jk_0 x' + jk_0 y' + jk_0 z' \right) \, dx' \, dy' \, dz'.
\]

(5)

Does the above equation remind you of anything?
Array Factor of a Uniformly Spaced Linear Array

\[ AF = \sum_n A_n \exp (j k x_n + j k y_n + j k z_n') \]

\[ = \sum_n A_n \exp (j k x_n') \]

\[ = \sum_n A_n \exp (j k x na) \] (6)
Array Factor of a Uniformly Spaced Planar Array

\[ AF = \sum_n A_n \exp (j k_x x_n' + j k_y y_n' + j k_z z_n') \]

\[ = \sum_n A_n \exp (j k_x x_n' + j k_y y_n') \]

\[ = \sum_p \sum_q A_{pq} \exp (j k_x x_{pq}' + j k_y y_{pq}') \]

\[ = \sum_p \sum_q A_{pq} \exp \left[ j k_x \left( pa + \frac{qb}{\tan \gamma} \right) + j k_y qb \right] \]  \hspace{1cm} (7)
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Continuous Linear Array

\[ AF(k_x) = \int_{-\infty}^{\infty} A(x') \exp(jk_x x') \, dx' \]
Discrete **Uniformly Spaced** Linear Array

\[
AF(k_x) = \sum_{n} A_n \exp(jk_x x'_n) = \sum_{n} A_n \exp(jk_x na)
\]
Discrete **Uniformly Spaced** Linear Array

Array Factor Linear Arrays Linear Arrays - Examples Planar Arrays Planar Arrays - Examples Synthesis Multiple Beam Arrays Problems

Discrete Uniformly Spaced Linear Array

\[ AF(k_x) = \sum_n A_n \exp(jk_x x'_n) = \sum_n A_n \exp(jk_x na) \]
Discrete **Uniformly Spaced** Linear Array

\[ AF(k_x) = \sum_{n} A_n \exp(jk_xx') = \sum_{n} A_n \exp(jk_xna) \]
Discrete Linear Array - **Progressive Phasing**

\[
AF (k_x - k_{x0}) = \sum_n A_n \exp \left[j \left(k_x - k_{x0}\right) x'_n\right] = \sum_n A_n \exp \left[j \left(k_x - k_{x0}\right) na\right] = \sum_n A_n \exp (-jk_{x0}na) \exp (jk_x na)
\]
Maximum Scan Limit

\[
    k_{x0} \leq \left( \frac{2\pi}{a} - k_0 \right) \\
    \Rightarrow k_0 \sin \theta_0 \leq \left( \frac{2\pi}{a} - k_0 \right) \\
    \Rightarrow \theta_0 \leq \sin^{-1} \left( \frac{2\pi}{ak_0} - 1 \right) \\
    \Rightarrow \theta_{0,\text{max}} = \sin^{-1} \left( \frac{2\pi}{ak_0} - 1 \right)
\]
Optimal Spacing

\[ k_{x0} \leq \left( \frac{2\pi}{a} - k_0 \right) \]

\[ k_0 \sin \theta_{0,\text{max}} \leq \left( \frac{2\pi}{a} - k_0 \right) \]

\[ a \leq \frac{1}{k_0} \left( \frac{2\pi}{1 + \sin \theta_{0,\text{max}}} \right) \]

\[ a_{\text{max}} = \frac{1}{k_0} \left( \frac{2\pi}{1 + \sin \theta_{0,\text{max}}} \right) \]
Beam Broadening

![Graph showing beam broadening](image)

**AF (in linear scale)**

- **Array Antennas - Theory & Design**
- CT531, DA-IICT
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1 Continuous Linear Array

Dipole Antenna of Arbitrary Length

\[
\begin{align*}
\mathbf{r} &= x\hat{x} + y\hat{y} + z\hat{z} \\
\mathbf{r}' &= z'\hat{z} \\
dz &= dz' \\
\end{align*}
\]

\[
I_0 \sin \left[ k_0 \left( \frac{1}{2} - z' \right) \right] \hat{z}, \quad 0 \leq z' \leq l/2
\]

\[
I_0 \sin \left[ k_0 \left( \frac{1}{2} + z' \right) \right] \hat{z}, \quad -l/2 \leq z' \leq 0
\]
1 Continuous Linear Array

Dipole Antenna of Arbitrary Length

Array factor of a continuous linear array antenna oriented along $z$ axis is given as

$$AF(k_z) = \int_{-1/2}^{+1/2} A(z') \exp(jk_z z') dz'$$

$$= \int_{-1/2}^{0} \left\{ I_0 \sin\left[k_0 \left(\frac{l}{2} + z'\right)\right]\right\} \exp(jk_z z') dz' + \int_{0}^{+1/2} \left\{ I_0 \sin\left[k_0 \left(\frac{l}{2} - z'\right)\right]\right\} \exp(jk_z z') dz'.$$

Also, we can prove the following integral identities (one can use Wolfram Online Integrator):

$$\int \sin\left[k_0 \left(\frac{l}{2} + z'\right)\right] \exp(jk_z z') dz' = e^{jk_z z'} \left\{ \frac{j k_z \sin\left[k_0 \left(\frac{l}{2} + 2z'\right)\right] - k_0 \cos\left[k_0 \left(\frac{l}{2} + 2z'\right)\right]}{k_0^2 - k_z^2} \right\}$$

and

$$\int \sin\left[k_0 \left(\frac{l}{2} - z'\right)\right] \exp(jk_z z') dz' = e^{jk_z z'} \left\{ \frac{j k_z \sin\left[k_0 \left(\frac{l}{2} - 2z'\right)\right] + k_0 \cos\left[k_0 \left(\frac{l}{2} - 2z'\right)\right]}{k_0^2 - k_z^2} \right\}$$
1 Continuous Linear Array

Dipole Antenna of Arbitrary Length

Using the integral identities provided in the previous slide gives

\[
\int_{-l/2}^{0} \left\{ \sin \left[ k_0 \left( \frac{l}{2} + z' \right) \right] \right\} \exp (jk_z z') \, dz' = \left[ \frac{jk_z \sin \left( \frac{k_0 l}{2} \right) - k_0 \cos \left( \frac{k_0 l}{2} \right)}{k_0^2 - k_z^2} \right] + e^{-jk_z \frac{l}{2}} \left[ \frac{k_0}{k_0^2 - k_z^2} \right]
\]

and

\[
\int_{0}^{+l/2} \left\{ \sin \left[ k_0 \left( \frac{l}{2} - z' \right) \right] \right\} \exp (jk_z z') \, dz' = e^{jk_z \frac{l}{2}} \left[ \frac{k_0}{k_0^2 - k_z^2} \right] - \left[ \frac{jk_z \sin \left( \frac{k_0 l}{2} \right) + k_0 \cos \left( \frac{k_0 l}{2} \right)}{k_0^2 - k_z^2} \right].
\]

So, array factor of a finite length dipole is given as

\[
AF = I_0 \left\{ \left( e^{jk_z \frac{l}{2}} + e^{-jk_z \frac{l}{2}} \right) \left[ \frac{k_0}{k_0^2 - k_z^2} \right] - \frac{2k_0 \cos \left( \frac{k_0 l}{2} \right)}{k_0^2 - k_z^2} \right\}
\]

\[
= I_0 \left\{ 2k_0 \times \left[ \frac{\cos \left( \frac{k_0 l}{2} \right) - \cos \left( \frac{k_0 l}{2} \right)}{k_0^2 - k_z^2} \right] \right\}. \tag{8}
\]
1 Continuous Linear Array

Dipole Antenna of Arbitrary Length

Finally, we need to multiply array factor with element pattern corresponding to a Hertzian dipole oriented along $z$ direction which is given readily in the below table.

<table>
<thead>
<tr>
<th>$E_{\theta}$</th>
<th>$\eta H_{\phi}$</th>
<th>$E_{\phi}$</th>
<th>$-\eta H_{\theta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_e = \hat{x}' \delta (x') \delta (y') \delta (z')$</td>
<td>$\kappa \cos \theta \cos \phi$</td>
<td>$-\kappa \sin \phi$</td>
<td></td>
</tr>
<tr>
<td>$J_e = \hat{y}' \delta (x') \delta (y') \delta (z')$</td>
<td>$\kappa \cos \theta \sin \phi$</td>
<td>$\kappa \cos \phi$</td>
<td></td>
</tr>
<tr>
<td>$J_e = \hat{z}' \delta (x') \delta (y') \delta (z')$</td>
<td>$-\kappa \sin \theta$</td>
<td>$0$</td>
<td></td>
</tr>
<tr>
<td>$J_m = \hat{x}' \delta (x') \delta (y') \delta (z')$</td>
<td>$-\left(\frac{\kappa \sin \phi}{\eta}\right)$</td>
<td>$-\left(\frac{\kappa \cos \theta \cos \phi}{\eta}\right)$</td>
<td></td>
</tr>
<tr>
<td>$J_m = \hat{y}' \delta (x') \delta (y') \delta (z')$</td>
<td>$\left(\frac{\kappa \cos \phi}{\eta}\right)$</td>
<td>$-\left(\frac{\kappa \cos \theta \sin \phi}{\eta}\right)$</td>
<td></td>
</tr>
<tr>
<td>$J_m = \hat{z}' \delta (x') \delta (y') \delta (z')$</td>
<td>$0$</td>
<td>$\left(\frac{\kappa \sin \theta}{\eta}\right)$</td>
<td></td>
</tr>
</tbody>
</table>

where, $\kappa = -\frac{j\eta k_0 e^{-j k_0 r}}{4\pi r}$ and $\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

So, far-field components of a dipole are given as

$$E_{\theta} = \eta H_{\phi} = I_0 \left\{ 2k_0 \times \left[ \frac{\cos \left(\frac{k_2 l}{2}\right) - \cos \left(\frac{k_0 l}{2}\right)}{k_0^2 - k_2^2} \right] \right\} \times \frac{j\eta k_0 e^{-j k_0 r}}{4\pi r} \sin \theta$$ (9)
2 Uniformly Spaced Discrete Linear Array (a=\(\frac{\lambda}{2}\))

Uniform Excitation
2 Uniformly Spaced Discrete Linear Array (a=\frac{\lambda}{2})

Uniform Excitation
2 Uniformly Spaced Discrete Linear Array (a=\lambda)

Uniform Excitation

Array Excitation

\[
\begin{array}{c}
\text{Array Excitation} \\
|A_u| \\
\end{array}
\]

Array – Factor in dB scale

\[
\begin{array}{c}
\text{Array – Factor in dB scale} \\
AP (u) \\
\end{array}
\]

u, where \( u = \sin \theta \) in the visible space
2 Uniformly Spaced Discrete Linear Array (a=\lambda)

Uniform Excitation

Normalized Factor in dB scale

Gain-Factor in dB scale
4 Uniformly Spaced Discrete Linear Array - **Scan** (a=\( \lambda \))

Uniform Excitation
5 Discrete Linear **End-fire Array** \( (a=\frac{\lambda}{2}) \)

Uniform Excitation

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Array Factor Linear Arrays Linear Arrays - Examples Planar Arrays Planar Arrays - Examples Synthesis Multiple Beam Arrays Problems

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CT531, DA-IICT
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Continuous Planar Array

Array placed in the $xy$ plane:

$$AF (k_x, k_y) = \iiint A (x', y') \exp (jk_x x' + jk_y y') \, dx' \, dy'$$

Array placed in the $yz$ plane:

$$AF (k_y, k_z) = \iiint A (y', z') \exp (jk_y y' + jk_z z') \, dy' \, dz'$$

Array placed in the $xz$ plane:

$$AF (k_x, k_z) = \iiint A (x', z') \exp (jk_x x' + jk_z z') \, dx' \, dz'$$
Visible Space in $k_xk_y$ domain

\[ k_x^2 + k_y^2 \leq k_0^2 \]
Discrete Uniformly Spaced Planar Array

\[ AF = \sum_p \sum_q A_{pq} \exp \left[ jk_x \left( pa + \frac{qb}{\tan \gamma} \right) + jk_y qb \right] \]
Grating Lobe Locations

\[ k_x a = 2 \mu \pi \quad \text{and} \quad \frac{k_x b}{\tan \gamma} + k_y b = 2 \nu \pi, \quad \text{where} \quad \mu, \nu = 0, \pm 1, \pm 2, \ldots \]
Typical Scanning Examples

\[ AB = \tan \theta_y \]
\[ AC = \tan \theta_x \]
Hemispherical Scanning
Hemispherical Scanning ... Contd
Hemispherical Scanning ... Contd

\[ AB = \tan \theta_3 \]

\[ x \parallel x'' \]
\[ y \parallel y'' \]

\[ \phi_1 = 90^\circ \]
\[ \phi_3 = 270^\circ \]
PAWE PAWS
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1 Planar Array (a=\lambda, b=\lambda)

Uniform Excitation
2 Planar Array \( (a=\frac{\lambda}{2}, b=\frac{\lambda}{2}) \)

Uniform Excitation
3 Planar Array \( (a=\frac{\lambda}{2}, b=\frac{\lambda}{2}) \) ... Scan

Uniform Excitation

![3D Graphs](image-url)
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Uniform Array - If $\theta^{\text{null}}$ is given

Array factor of a $M$ element linear array oriented along $x$ axis is given as

$$AF(k_x) = \frac{\sin\left(\frac{k_x Ma}{2}\right)}{\sin\left(\frac{k_x a}{2}\right)}.$$

So, first null position is given from the below equation:

$$\frac{k_{x1}^{\text{null}} Ma}{2} = \pi.$$

If null-to-null beamwidth, $2\theta^{\text{null}}$, is given, then

$$\Rightarrow M = \frac{2\pi}{k_{x1}^{\text{null}} a}$$

$$\Rightarrow M = \frac{2\pi}{k_0 \sin \theta^{\text{null}} a}.$$  \hspace{1cm}(10)
Binomial Array

For binomial arrays, array factor is given as

\[ AF(k_x) = \left( e^{jk_xa} - e^{jk_{x0}a} \right)^{M-1}. \]  

(11)

Reasons for choosing the above array factor are:

- array factor should be a periodic function in \(k_x\) domain with a period of \(\frac{2\pi}{a}\)
- array factor should have an order of \(M - 1\) in \(e^{jk_xa}\) domain
- all the zeros should present at one single location, i.e., \(k_x = k_{x0}\), so that array factor will not have any side-lobes

One can obtain binomial array coefficients by equating (11) to the definition of array factor

\[ AF(k_x) = \sum A_m e^{jk_{x}ma}. \]
Dolph-Chebyshev Array

Array factor corresponding to Dolph-Chebyshev array is given as

\[
AF (k_x) = T_{M-1} \left( -c \cos \frac{k_x a}{2} \right)
\] (12)

where Chebyshev polynomial \( T_N (x) \) is defined as

\[
T_N (x) = \begin{cases} 
\cos (N \cos^{-1} x), & |x| \leq 1 \\
\cosh (N \cosh^{-1} x), & |x| > 1 
\end{cases}
\] (13)

The parameter \( c \) is decided by the given SLR. If the given SLR is \( R \) in linear scale, then \( c \) is decided such that

\[
T_{M-1} (c) = R \Rightarrow \cosh \left[ (M - 1) \cosh^{-1} c \right] = R \Rightarrow c = \cosh \left( \frac{\cosh^{-1} R}{M - 1} \right). 
\] (14)
Dolph-Chebyshev Array Mapping ... Even Order

\[ AF(k_x) = T_{M-1} \left( -c \cos \frac{k_x a}{2} \right) \]
Dolph-Chebyshev Array Mapping ... Odd Order

\[ AF(k_x) = T_{M-1} \left( -c \cos \frac{k_x a}{2} \right) \]
Dolph-Chebyshev Array ... Contd

Reasons for choosing array factor given by (12) are as given below:

- array factor should be a periodic function in $k_x$ domain with a period of $\frac{2\pi}{a}$ ($\frac{4\pi}{a}$) for odd (even) numbered arrays
- array factor should have an order of $M - 1$ in $e^{jka}$ domain
- $[-c, +c]$ region of Chebyshev polynomial should get mapped to $[0, \frac{2\pi}{a}]$ region of $k_x$ domain as shown in the previous mapping figures.
**Dolph-Chebyshev Array ... Array Factor Zeros**

Array factor zeros are given as shown below:

\[
T_{M-1} \left( -c \cos \frac{k_{x,\text{null}} a}{2} \right) = 0
\]

\[
\Rightarrow \cos \left[ (M - 1) \cos^{-1} \left( -c \cos \frac{k_{x,\text{null}} a}{2} \right) \right] = 0
\]

\[
\Rightarrow (M - 1) \cos^{-1} \left( -c \cos \frac{k_{x,\text{null}} a}{2} \right) = \pm (2n - 1) \frac{\pi}{2}, \text{ where } n = \pm 1, \pm 2, \pm 3, \ldots
\]

\[
\Rightarrow k_{x,\text{null}} a = 2 \cos^{-1} \left\{ \frac{1}{c} \cos \left[ \left( \frac{2n - 1}{M - 1} \right) \frac{\pi}{2} \right] \right\}.
\]

So, array factor can be written in terms of zeros as

\[
AF(k_x) = \prod \left( e^{jk_x a} - e^{j k_{x,\text{null}} a} \right).
\]
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Multiple Beam Phased Array Antenna System

Radiating elements

Power combiner

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Multiple Beams with Uniform Excitation

Radiating elements

Power combiner

Power combiner
Multiple Beams - Orthogonality

![Graph showing multiple beams and their orthogonality.](image-url)
Multiple Beams - Orthogonality
Multiple Beams - Orthogonality

\[ a_n = \sum_{k=0}^{K-1} u_k e^{-jn\psi_k} \]
Multiple Beams - Orthogonality

\[ a_n = \sum_{k=0}^{K-1} v_k e^{-jn\psi_k} \]

\[ K = N \]

\[ \psi_k = \psi_0 + \frac{2\pi k}{N} \]
FFT Algorithm

\[ a_n = \sum_{k=0}^{N-1} v_k e^{-jn \frac{2\pi k}{N}} \]

\[ = \sum_{m=0}^{N/2-1} v_{2m} e^{-jn \frac{2\pi m}{N}} + \sum_{m=0}^{N/2-1} v_{2m+1} e^{-jn \frac{2\pi (2m+1)}{N}} \]

\[ = \sum_{m=0}^{N/2-1} v_{2m} e^{-jn \frac{2\pi m}{N/2}} + e^{-jn \frac{\pi}{N}} \sum_{m=0}^{N/2-1} v_{2m+1} e^{-jn \frac{2\pi m}{N/2}} \]

\[ = P_n + W_N^n Q_n \]

\[ a_0 = P_0 + W_N^0 Q_0 \]
\[ a_1 = P_1 + W_N^1 Q_1 \]
\[ a_2 = P_2 + W_N^2 Q_2 \]
\[ \vdots \]
\[ a_{N/2-1} = P_{N/2-1} + W_N^{N/2-1} Q_{N/2-1} \]
\[ a_{N/2} = P_0 - W_N^0 Q_0 \]
\[ a_{N/2+1} = P_1 - W_N^1 Q_1 \]
\[ \vdots \]
\[ a_{N-1} = P_{N/2-1} - W_N^{N/2-1} Q_{N/2-1} \]
FFT Algorithm ... Contd

\[ P_n = \sum_{m=0}^{N/2-1} v_{2m} e^{-jn \frac{2\pi m}{N/2}} \]

\[ Q_n = \sum_{m=0}^{N/2-1} v_{2m+1} e^{-jn \frac{2\pi m}{N/2}} \]

\[ P_n = U_n + W_n^{n/2} V_n \]

\[ P_{n+\frac{N}{4}} = U_n - W_n^{n/2} V_n \]

\[ W_{\tau N} = W_N^n \]

\[ Q_n = Y_n + W_n^{n/2} Z_n \]

\[ Q_{n+\frac{N}{4}} = Y_n - W_n^{n/2} Z_n \]

\[ P_n = U_n + W_N^{2n} V_n \]

\[ P_{n+\frac{N}{4}} = U_n - W_N^{2n} V_n \]
Example: 8 Point FFT
4 Beam Array

\[ \begin{align*}
&v_0 \\
&v_2 \\
&v_1 \\
&v_3 \\
\end{align*} \quad \begin{align*}
&\text{Radiating elements} \\
&a_0 \\
&a_1 \\
&a_2 \\
&a_3 \\
\end{align*} \quad \begin{align*}
&z, z' \\
&x, x' \\
\end{align*} \]
4 Beam Array ... Highlighting Butterfly Section
Butterfly Implementation in Analog Domain
So, Finally 4 Beam Analog Array is ...
Outline

1. Array Factor
2. Linear Arrays
3. Linear Arrays - Examples
4. Planar Arrays
5. Planar Arrays - Examples
6. Synthesis
7. Multiple Beam Arrays
8. Problems
Linear Arrays - I

1. Calculate the array factor of a continuous linear array oriented along $z'$ axis and having an aperture distribution

\[
A(z') = \begin{cases} 
1, & |z'| \leq \frac{L}{2} \\
0, & \text{elsewhere}.
\end{cases}
\]

Also calculate (a) the directions in which array factor exhibits zero radiations (i.e., null directions), and (b) null-to-null beamwidth.

2. Calculate the array factor of a discrete uniformly spaced linear array oriented along $x'$ axis and having uniform aperture distribution. Assume that the array is having $M$ number of elements ($M$ can be either even or odd) and the spacing between any two consecutive elements is $a$. Also calculate (a) the directions in which array factor exhibits zero radiations (i.e., null directions), and (b) null-to-null beamwidth.

3. Prove that array factor of any uniformly spaced odd numbered array, located symmetrically with respect to origin will be periodic in $k_x$ domain with a periodicity of $\frac{2\pi}{a}$. Also prove that array factor of any even numbered array located symmetrically with respect to origin will be periodic in $k_x$ domain with a periodicity of $\frac{4\pi}{a}$. 
For a given linear array oriented along $x$ axis and having an uniform spacing of 15mm, calculate the progressive phase shift required to scan the beam from broad-side direction (i.e., $\theta = 0^\circ$) to (a) $\theta_0 = 30^\circ$ (b) $\theta_0 = 90^\circ$. Assume that the operating frequency of the array is 10 GHz.

For a given linear array oriented along $x$ axis and having an uniform spacing of $a = 0.75\lambda$, calculate the maximum scanning that can be done so that grating-lobe doesn’t enter into the visible-space.

If the main beam corresponding to a given linear array has to be scanned to a maximum angle of $\theta_0 = 40^\circ$, then what should be the maximum possible uniform spacing between consecutive elements.
Planar Arrays

1. Calculate the array factor of a discrete uniformly spaced planar array (assume rectangular lattice) placed in the $x'y'$ plane and having uniform aperture distribution along both $x'$ and $y'$ axes. Assume that number of elements and spacing along $x'$ axis are $M$ and $a$, respectively. Similarly, Assume that number of elements and spacing along $y'$ axis are $N$ and $b$, respectively.

2. For a given planar array in $xy$ axis and having $a = 15\text{mm}$ and $b = 20\text{mm}$, calculate the progressive phase shifts required to scan the beam from broad-side direction (i.e., $\theta = 0^\circ$) to (a) $\theta_0 = 45^\circ$ and $\phi_0 = 45^\circ$. Assume that the operating frequency of the array is 10 GHz.
Array Synthesis

1. Calculate the number of elements required to obtain an array factor with 10° null-to-null beamwidth. Assume that the array elements are uniformly excited and $a = \lambda / 2$.

2. Design a 5-element Binomial array having all the zeros at $\theta_0 = 30°$ and having an uniform spacing of $a = \lambda / 2$.

3. Design a 5-element Dolph-Chebyshev array having SLR of 30 dB and spacing $a = \lambda / 2$. 
Multiple Beam Arrays

1. Develop the 16 point FFT algorithm and construct its signal flow graph.

2. For an 8 element array forming 8 beams (assume that the beams are symmetric with respect to the broadside direction), calculate the **progressive phase shifts** required for forming the beams. Also calculate the corresponding **excitation values for each array element** when all the beams are simultaneously excited.
References
