Outline

1. Introduction
2. Random Variables
3. Random Processes
4. Noise Characterization
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4. Noise Characterization
Noise

Noise can be internal too...

Noise is the undesired signal that gets added to the desired signal and reaches the destination. Noise ultimately determines the threshold for the minimum signal that can be reliably detected by a receiver.
Noise can be internal too ...
Noise is the **undesired signal** that gets added to the desired signal and reaches the destination.
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Noise ultimately determines the threshold for the minimum signal that can be reliably detected by a receiver.
Noise

Noise lives with the desired signal. Neither amplification nor the filtering can alleviate the effect of noise on the desired signal. The only way to keep away from the effects of noise is to see that less amount of noise, relative to the desired signal, is present at the destination.
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External Noise - Sources

• Noise from stars including the sun (20MHz – 1.5GHz)
• Lightning (2MHz – 10MHz)
• Thermal noise from the ground
• Cosmic background noise from the sky
• Man made noise, e.g., spark plugs, engine noise, etc (1MHz – 500MHz)
• Radio, TV, and cellular stations
• Wireless devices
• Microwave ovens
• Deliberate jamming devices
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- **Deliberate** jamming devices
Internal Noise - Sources

• Thermal noise is the most basic type of noise, being caused by thermal vibration of bound charges. It is also known as Johnson or Nyquist noise.
• Shot noise is due to random fluctuations of charge carriers in an electron tube or solid-state device.
• Flicker noise occurs in solid-state components and vacuum tubes. Flicker noise power varies inversely with frequency, and so is often called 1/f noise.
• Plasma noise is caused by random motion of charges in an ionized gas, such as a plasma, the ionosphere, or sparking electrical contacts.
• Quantum noise results from the quantized nature of charge carriers and photons; it is often insignificant relative to other noise sources.
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- **Quantum noise** results from the quantized nature of charge carriers and photons; it is often insignificant relative to other noise sources.
In some cases, such as radiometers or radio astronomy systems, the desired signal is actually the noise power received by an antenna, and it is necessary to distinguish between the received noise power and the undesired noise generated by the receiver system itself.
As you have seen, there are many types of noise sources. However, we will concentrate more on thermal noise. Can you guess the reason?
Before proceeding further, let’s **recap** the theory of random variables and processes ...
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Introduction

Random Variables

Random variable

Outcome of a random experiment could be numerical or non-numerical.

Non-numerical example: coin tossing experiment – Head or Tail.

For non-numerical outcomes, one can assign certain numbers.

We may assign 1 for Head and -1 for Tail.

When such numerical values are assigned to a variable, the variable is called a random variable.

For the coin tossing experiment, variable $x$ can take either 1 or -1 depending on the outcome.

Probability of a random variable $x$ taking values $x_i$ is $P_x(x_i)$.

Discrete random variable: If $\{x_i\}$ are distinct, then $x$ is a discrete random variable, such that $\sum_i P_x(x_i) = 1$. 

Random Variables, Processes, and Noise

Communication Systems, Dept. of EEE, BITS Hyderabad
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**Discrete random variable**: If $\{x_i\}$ are **distinct**, then $x$ is a discrete random variable, such that

$$\sum_i P_x(x_i) = 1.$$
Continuous Random Variables

Continuous RV can assume any value in a given interval. The probability density function (PDF) is the appropriate definition for continuous random variables. Probability for a continuous RV is defined in terms of PDF as

\[
P(x_1 \leq x \leq x_2) = \int_{x_1}^{x_2} p(x) \, dx.
\]

Of course,

\[
\int_{-\infty}^{\infty} p(x) \, dx = 1.
\]

And cumulative distribution function (CDF) is defined as

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F(x) = P(x \leq x) = \int_{-\infty}^{x} p(x) \, dx.
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Of course,

$$\int_{-\infty}^{\infty} p_x (x) \, dx = 1. \quad (2)$$

And cumulative distribution function (CDF) is defined as

$$F_x (x) = P (x \leq x) = \int_{-\infty}^{x} p_x (x) \, dx. \quad (3)$$
Statistical Averages – Means

For a continuous RV case, the mean is given by

$$\bar{x} = \mathbb{E}[x] = \int_{-\infty}^{\infty} x p(x) \, dx.$$ (4)

Mean of a function \(y = g(x)\) of a random variable is

$$g(x) = \mathbb{E}[g(x)] = \int_{-\infty}^{\infty} g(x) p(x) \, dx.$$ (5)

The above expression can be generalized for two random variables is of a random variable as

$$g(x, y) = \mathbb{E}[g(x, y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) p_{xy}(x, y) \, dx \, dy.$$ (6)
Statistical Averages – Means

For a continuous RV case, the mean is given by

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Mean of a function \((y = g(x))\) of a random variable is

\[ \overline{g(x)} = E[g(x)] = \int_{-\infty}^{\infty} g(x) p_x(x) \, dx. \] (5)
Statistical Averages – Means

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Statistical Averages – Moments

The $n$th moment of a random variable $x$ is defined as

$$
\hat{x}^n = \int_{-\infty}^{\infty} x^n p_x(x) \, dx.
$$

Similarly, the $n$th central moment of a random variable $x$ is defined as

$$
(\overline{x}^n = \int_{-\infty}^{\infty} (x - \overline{x})^n p_x(x) \, dx.
$$

The second central moment of an RV $x$ is called variance and denoted by

$$
\sigma^2_x = (x - \overline{x})^2 = x^2 - 2\overline{x}x + \overline{x}^2 = x^2 - \overline{x}^2.
$$
Statistical Averages – Moments

The *nth moment* of a random variable $x$ is defined as

$$
\bar{x}^n = \int_{-\infty}^{\infty} x^n p_x(x) \, dx.
$$

(7)
Statistical Averages – Moments

The \textit{nth moment} of a random variable \( x \) is defined as

\[ \overline{x^n} = \int_{-\infty}^{\infty} x^n p_x (x) \, dx. \]  \hfill (7)

Similarly, the \textit{nth central moment} of a random variable \( x \) is defined as

\[ (x - \overline{x})^n = \int_{-\infty}^{\infty} (x - \overline{x})^n p_x (x) \, dx. \]  \hfill (8)
Statistical Averages – Moments

The \textit{\textbf{\text{}nth moment}} of a random variable $x$ is defined as

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$$\overline{(x - \bar{x})^n} = \int_{-\infty}^{\infty} (x - \bar{x})^n p_x(x) \, dx.$$  \hfill (8)

The second central moment of an RV $x$ is called \textit{\textbf{\text{}variance}} and denoted by $\sigma_x^2$, where $\sigma_x$ is known as \textit{\textbf{\text{}standard deviation}}.

$$\sigma_x^2 = \overline{(x - \bar{x})^2}$$
$$= x^2 - 2\bar{x}^2 + \bar{x}^2 = \bar{x}^2 - \bar{x}^2$$ \hfill (9)
Statistical Averages – Corollary
Statistical Averages – Corollary

If $x$ and $x$ are independent RVs and

$$z = x + y,$$  \hspace{1cm} (10)
Statistical Averages – Corollary

If $x$ and $y$ are independent RVs and

$$z = x + y,$$  \hspace{1cm} (10)

then

$$\bar{z} = \bar{x} + \bar{y}$$  \hspace{1cm} (11)

$$\sigma_z^2 = \sigma_x^2 + \sigma_y^2.$$  \hspace{1cm} (12)
Central-Limit Theorem
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If $x$ and $x'$ are independent RVs and

$$z = x + y,$$ (13)
Central-Limit Theorem

If $x$ and $x$ are independent RVs and

$$z = x + y,$$ 

(13)

then

$$p_z(z) = \int_{-\infty}^{\infty} p_x(x) p_y(z - x) \, dx.$$ 

(14)

From the above equation, it is clear that the PDF $p_z(z)$ is the convolution of PDFs $p_x(x)$ and $p_y(y)$. This result can be extended to $n$ number of RVs.
Central-Limit Theorem

If \( x \) and \( y \) are independent RVs and

\[
z = x + y, \quad (13)
\]

then

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p_z(z) = \int_{-\infty}^{\infty} p_x(x) p_y(z-x) \, dx. \quad (14)
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From the above equation, it is clear that the PDF \( p_z(z) \) is the convolution of PDFs \( p_x(x) \) and \( p_y(y) \). This result can be extended to \( n \) number of RVs.

Under certain conditions, the sum of large number of independent random variables tends to be a Gaussian random variable, independent of the PDFs of the random variables involved. This is the central-limit theorem.
Central-Limit Theorem – Demonstration
Central-Limit Theorem – Demonstration
Central-Limit Theorem – Demonstration

\[ p_x(x) \]

\[ p_x(x) \ast p_x(x) \]
Central-Limit Theorem – Demonstration

$p_x(x)$

$-1 \quad 1$

$x$

$p_x(x) \ast p_x(x)$

$-2 \quad 2$

$x$

$p_x(x) \ast p_x(x) \ast p_x(x)$

$-3 \quad 3$

$x$
The Gaussian (Normal) PDF

The probability density function (PDF) of a Gaussian random variable is given by:

$$f(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where $\mu$ is the mean and $\sigma$ is the standard deviation.

The cumulative distribution function (CDF) is given by:

$$F(x; \mu, \sigma) = \frac{1}{2} \left[1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$$

where $\operatorname{erf}$ is the error function.
The Gaussian (Normal) PDF

\[ p_x (x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \]

\[ F_x (x) = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = 1 - Q \left( \frac{x - \mu}{\sigma} \right) \]
The Gaussian (Normal) PDF

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The Gaussian (Normal) PDF – Interpretation
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Random Process
Random Process

\[ x(t, \zeta_1) \]
Random Process

\[
x(t, \zeta_1)
\]
\[
x(t, \zeta_2)
\]
Random Process
Random Process

\[ x(t, \zeta_1) \]

\[ x(t, \zeta_2) \]

\[ x(t, \zeta_3) \]

\[ x(t, \zeta_4) \]
Random Process

Sample function

$t$

Sample function

$t$

Sample function

$t$

Sample function

$t$

$x(t, \zeta_1)$

$x(t, \zeta_2)$

$x(t, \zeta_3)$

$x(t, \zeta_4)$
Random Process

$x(t, \zeta_1)$

$x(t, \zeta_2)$

$x(t, \zeta_3)$

$x(t, \zeta_4)$

Ensemble
Random Process

$x(t_1) = x_1$

$x(t, \zeta_1)$

Random variable

$x(t, \zeta_2)$

$x(t, \zeta_3)$

$x(t, \zeta_4)$
Random Process

Random variable $x$ is a function of time $t$. For different values of $t$, the random variable $x(t)$ takes different realizations, denoted by $x(t_1) = x_1, x(t_2) = x_2, x(t_3), x(t_4)$. Each realization is a possible outcome of the random variable at a given time.
Random Process

A random variable that is a function of time is called a random process or stochastic process. In other words, a random process is just a collection of an infinite number of RVs, which are generally dependent. So, a random process is completely described by the joint PDF $p_{x_1 x_2 \cdots x_n}(x_1, x_2, \ldots, x_n)$ which can also be expressed as $p_{x_1 x_2 \cdots x_n}(x_1, x_2, \ldots, x_n; t_1, t_2, \ldots, t_n)$ or simply $p_{x}(x; t)$.
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\[ p_{x_1x_2\cdots x_n}(x_1, x_2, \ldots, x_n) \]
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or simply

\[ p_x (x; t) \]
Random Process ... A Few More Things
We can always derive a lower order PDF from a higher order PDF by integration. For instance,

\[ p_{x_1}(x_1) = \int_{-\infty}^{\infty} p_{x_1x_2}(x_1, x_2) \, dx_2. \] (15)
Random Process ... A Few More Things

We can always derive a lower order PDF from a higher order PDF by integration. For instance,
\[ p_{x_1}(x_1) = \int_{-\infty}^{\infty} p_{x_1x_2}(x_1, x_2) \, dx_2. \] (15)

The mean \( \bar{x}(t) \) of a random process \( x(t) \) can be determined from the first-order PDF as
\[ \bar{x}(t) = \int_{-\infty}^{\infty} xp_x(x; t) \, dx. \] (16)
A Few Other Types of Random Process
A Few Other Types of Random Process

\[ x(t, \zeta_1) \]
\[ x(t, \zeta_2) \]
\[ x(t, \zeta_3) \]
\[ x(t, \zeta_4) \]
A Few Other Types of Random Process

\[ x(t, \zeta_1) \]
\[ x(t, \zeta_2) \]
\[ x(t, \zeta_3) \]
\[ x(t, \zeta_4) \]
Auto-Correlation of a Random Process
Auto-Correlation of a Random Process

\[ R_{x}(t_1, t_2) = \int_{-\infty}^{\infty} x(t_1) x(t_2) \, dx_1 \, dx_2 \]
Auto-Correlation of a Random Process

\[
R_{x}(t_1, t_2) = \mathbb{E}[x(t_1)x(t_2)] = \mathbb{E}[x_1x_2] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_1x_2 p_{x_1x_2}(x_1, x_2) \, dx_1 \, dx_2
\]
Auto-Correlation of a Random Process

Let's consider two random processes, $x(t)$ and $y(t)$, with auto-correlation functions $R_x(\tau)$ and $R_y(\tau)$ respectively. The auto-correlation function measures the similarity between the process and a time-shifted version of itself. It is defined as:

$$R_x(\tau) = \mathbb{E}[x(t_1) x(t_2)]$$

where $\mathbb{E}$ denotes the expectation operator.

Similarly, for $y(t)$:

$$R_y(\tau) = \mathbb{E}[y(t_1) y(t_2)]$$

These functions are important in signal processing and communication theory as they provide insights into the temporal behavior of random processes.
Auto-Correlation of a Random Process

\[ R_x(\tau) = \int_{-\infty}^{\infty} f(x_1, x_2) x_1 x_2 \, dx_1 \, dx_2 \]

\[ R_y(\tau) = \int_{-\infty}^{\infty} f(y_1, y_2) y_1 y_2 \, dy_1 \, dy_2 \]
Stationary Random Process
Stationary Random Process

A random process whose statistical characteristics do not change with time is classified as a stationary random process.
Stationary Random Process

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Stationary Random Process
Stationary Random Process

\[ p_x(x; t) = p_x(x) \quad (17) \]
Stationary Random Process

\[ p_x(x; t) = p_x(x) \]  

(17)

\[ R_x(t_1, t_2) = R_x(t_2 - t_1) \]  

(18)
Stationary Random Process

\( p_x (x; t) = p_x (x) \)  \hspace{1cm} (17)

\( R_x (t_1, t_2) = R_x (t_2 - t_1) \)  \hspace{1cm} (18)

\( R_x (\tau) = \bar{x} (t) \bar{x} (t + \tau) \)  \hspace{1cm} (19)
Stationary Random Process

\[ p_x (x; t) = p_x (x) \]  \hspace{1cm} (17)

\[ R_x (t_1, t_2) = R_x (t_2 - t_1) \]  \hspace{1cm} (18)

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A process is called stationary process only when first-order as well as all the higher order PDFs, such as \( p_{x_1 x_2 \ldots x_n} (x_1, x_2, \ldots, x_n) \) are all independent of the choice of origin.
Wide-Sense Stationary Random Process
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This means

$$\bar{x}(t) = \text{constant} \quad (20)$$

and

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\]

Such a process is known as wide-sense stationary, or weakly stationary process.
Ergodic Random Process

In ergodic process, ensemble averages are equal to the time averages of any sample function. Thus for an ergodic process \( x(t) \),

\[
x(t) = \bar{x}(t)
\]

where

\[
\bar{x}(t) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) \, dt
\]

and

\[
R_x(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) x(t+\tau) \, dt
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An Ergodic process is necessarily a stationary process; but the converse is not true.
Ergodic Random Process

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\]
In ergodic process, ensemble averages are equal to the time averages of any sample function. Thus for an ergodic process $x(t)$,

\[
\overline{x(t)} = \tilde{x(t)}
\]

\[
R_x(\tau) = R_x(\tau),
\]

where

\[
\tilde{x(t)} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) \, dt
\]

and

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R_x(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) x(t + \tau) \, dt.
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An Ergodic process is necessarily a stationary process; but the converse is not true.
Why Ergodic Random Processes Notion is Important?

Because many of the stationary processes encountered in practice are ergodic with respect to at least second-order averages, i.e., with respect to mean and auto-correlation values. And we need only the first- and second-order averages when we are dealing with linear systems.
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Classification of Random Processes
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Random process

Wide-sense stationary
Classification of Random Processes
Classification of Random Processes

- Random process
- Wide-sense stationary
- Stationary
- Ergodic
PSD of a Random Process

PSD of a random process $x(t)$ is given as

$$S_x(\omega) = \lim_{T \to \infty} \left[ |X_T(\omega)|^2 T \right], \quad (22)$$

where $X_T(\omega)$ is the Fourier transform of the truncated random process $x(t) \text{rect}(t/T)$.

PSD is related to auto-correlation function as

$$S_x(\omega) = \hat{\infty}^{-\infty} R_x(\tau) e^{-j\omega \tau} d\tau \quad (23)$$

where $R_x(\tau) = x^*(t) x(t+\tau)$.

The average power of a wide-sense random process $x(t)$ is its mean square value $P_x = \mathbb{E}[x^2(t)] = R_x(0) = \frac{1}{2\pi} \hat{\infty}^{-\infty} S_x(\omega) d\omega. \quad (24)$
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$$P_x = \overline{x^2} = R_x(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_x(\omega) d\omega.$$

(24)
Transmission of Random Processes Through Linear Systems
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\[ x(t) \rightarrow H(\omega) \rightarrow y(t) \]

\[ R_y(\tau) = h(\tau) \ast h(-\tau) \ast R_x(\tau) \]  \hspace{2cm} (25)

\[ S_y(\omega) = |H(\omega)|^2 S_x(\omega) \]  \hspace{2cm} (26)
Transmission of Random Processes Through Linear Systems

\[ R_y(\tau) = h(\tau) * h(-\tau) * R_x(\tau) \] (25)
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\[ x(t) \xrightarrow{H(\omega)} y(t) \]

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\[ S_y(\omega) = |H(\omega)|^2 S_x(\omega) \]  \hspace{1cm} (26)
Sum of Random Processes

If two stationary processes (at least in the wide sense) $x(t)$ and $y(t)$ are added to form a process $z(t)$, i.e.,

$$z(t) = x(t) + y(t),$$

then

$$R_z(\tau) = R_x(\tau) + R_y(\tau) + 2\alpha_z \gamma.$$  (27)

Most processes of interest in communication problems have zero means. So, if $x(t)$ and $y(t)$ are uncorrelated with either $x = 0$ or $y = 0$

$$R_z(\tau) = R_x(\tau) + R_y(\tau)$$  (28)

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Outline

1. Introduction
2. Random Variables
3. Random Processes
4. Noise Characterization
As we have seen, there are many types of noise sources. However, we will concentrate more on thermal noise for the reasons mentioned earlier. So, let’s study about thermal noise in this section.
Random motions of electrons produce small, random voltage fluctuations at the resistor terminals, which have a zero average value but a nonzero root mean square (rms) value given by Planck's blackbody radiation law,

$$v_n = \sqrt{\frac{4hfB}{kT}} e^{hf/(kT)} - 1, \quad (30)$$

where $h$ and $k$ are Plank's and Boltzmann's constants, respectively.
Thermal (Johnson-Nyquist) Noise

Random motions of electrons produce small, random voltage fluctuations at the resistor terminals, which have a zero average value but a nonzero root mean square (rms) value given by Planck's blackbody radiation law,

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Rayleigh–Jeans Approximation

At microwave frequencies, where $hf \ll kT$, (30) can be simplified to

$$V_n = \sqrt{\frac{4kTBR}{}}. \quad (31)$$

For very high frequencies or very low temperatures, however, this approximation may be invalid, in which case (30) should be used.

From the equation (31), it is evident that thermal noise is independent of frequency. So, thermal noise is a white noise.
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Maximum Available Noise Power

The power delivered to the load shown in the above figure is

\[ P_n = \frac{V_n^2}{R} = kT \frac{B}{R}, \quad (32) \]

since \( V_n \) is an rms voltage. This important result gives the maximum available noise power from the noisy resistor at temperature \( T \).
Maximum Available Noise Power

Power delivered to the load shown in the above figure is
\[ P_n = \frac{(V_n^2)}{2R} = kTB, \quad (32) \]
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Independent white noise sources can be treated as Gaussian-distributed random variables, so the noise powers (variances) of independent noise sources are additive.
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Independent white noise sources can be treated as Gaussian-distributed random variables, so the noise powers (variances) of independent noise sources are additive.
Thermal Noise - Some Observations

• As $B \to 0$, $P_n \to 0$. This means that systems with smaller bandwidths collect less noise power.

• As $T \to 0$, $P_n \to 0$. This means that cooler devices and components generate less noise power.

• As $B \to \infty$, $P_n \to \infty$. This is the so-called ultraviolet catastrophe, which does not occur in reality because Rayleigh-Jeans approximation is valid only when $hf \ll kT$. 
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- As $B \to \infty$, $P_n \to \infty$. This is the so-called ultraviolet catastrophe, which does not occur in reality because Rayleigh-Jeans approximation is valid only when $hf \ll kT$. 
The characterization of noise effects in communication systems in terms of noise temperature and noise figure will apply to all types of noise, regardless of the source, as long as the spectrum of the noise is relatively flat over the bandwidth of the system.
Equivalent Noise Temperature

If an arbitrary source of noise (thermal or non-thermal) is white, it can be modeled as an equivalent thermal noise source, and characterized with an equivalent noise temperature, $T_e$. $T_e = N_0/k_B$. (33)
Equivalent Noise Temperature

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$$T_e = \frac{N_o}{kB}$$
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$$T_e = \frac{N_0}{kB}$$

(33)
Equivalent Noise Temperature of a Noisy Amplifier
Equivalent Noise Temperature of a Noisy Amplifier

\[ T_e = \frac{N_0}{GkB} \]  

(34)
Noise Figure

Noise figure, $F$, is a measure of this reduction in signal-to-noise ratio, and is defined as

$$F = \frac{S_i}{N_i} = \frac{S_o}{N_o} \geq 1. \quad (35)$$

By definition, the input noise power is assumed to be the noise power resulting from a matched resistor at $T_0 = 290K$, i.e., $N_i = kT_0B$. 

Noisy network $G, B, T_e$.
Noise Figure

Noisy network \( G, B, T_e \)

\[ R \]

\[ T_0 = 290 \text{ K} \]

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Relation Between $F$ and $T_e$

\[ F = \frac{S_i}{S_o} \times \frac{N_o}{N_i} = \frac{1}{G} \times \frac{Gk (T_0 + T_e) B}{kT_0 B} = 1 + \frac{T_e}{T_0} \geq 1. \]
Equivalent Noise Temperature of a Cascaded System

The noise power at the output of the second stage is

\[ N_o = G_2 N_1 + G_2 kT_2 B = G_2 G_1 k (T_0 + T_{e1}) B + G_2 kT_2 B. \]

For the equivalent system we have

\[ N_o = G_1 G_2 k (T_{e,\text{cas}} + T_0) B. \]

So, comparing the above equations gives

\[ T_{e,\text{cas}} = T_{e1} + T_{e2} G_1. \]
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Noise Figure of a Cascaded System

\[
T_0(\text{\textit{F}}_{\text{\textit{cas}}} - 1) = T_0(\text{\textit{F}}_1 - 1) + T_0(\text{\textit{F}}_2 - 1) \quad \text{G}_1
\Rightarrow \text{\textit{F}}_{\text{\textit{cas}}} = \text{\textit{F}}_1 + \text{\textit{F}}_2 - 1 \quad \text{G}_1.
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\[ \Rightarrow F_{\text{cas}} = F_1 + \frac{F_2 - 1}{G_1}. \]  \hspace{1cm} (38)
Generalization
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$$F_{\text{cas}} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1G_2} + \cdots.$$  \hspace{1cm} (40)
Lossy Line held at Physical Temperature $T$
Consider a lossy line (lossy lines or networks containing passive elements can generate only thermal noise) with a matched source resistor that is also at temperature $T$ as shown in the figure.

The power gain, $G$, of a lossy line is less than unity; the loss factor, $L$, can be defined as $L > 1$.

Input noise power is $kTB$. Because the entire system is in thermal equilibrium at the temperature $T$, and thermal noise power is independent of the resistance value, the output noise power also must be $N_o = kTB$. 

- $T$ is the physical temperature.
- $R$ is the thermal noise resistance.
- $N_i = kTB$ is the input noise power.
- $L, T, Z_o = R$ are the loss factor, temperature, and characteristic impedance, respectively.
- $N_o = kTB$ is the output noise power.
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Thus, we have

$$N_0 = kTB = GkTB + GN_{\text{added}}$$

where $N_{\text{added}}$ is the noise generated by the line, as if it appeared at the input terminals of the line.
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Thus, we have

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$$N_{\text{added}} = kTB \frac{1 - G}{G} = kTB \left( L - 1 \right).$$
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From the above equation, it is clear that the **equivalent noise temperature** of the lossy line is

$$T_e = (L - 1) T.$$
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So, **noise figure** of the lossy line is

$$F = 1 + \frac{T_e}{T_0} = 1 + (L - 1) \frac{T}{T_0}.$$
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$$T_e = (L - 1) T.$$ 

So, noise figure of the lossy line is

$$F = 1 + \frac{T_e}{T_0} = 1 + (L - 1) \frac{T}{T_0}.$$ 

When $T = T_0$, for lossy networks, $F = L$. 